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AN IMPROVED CONSTITUTIVE STATISTICAL DAMAGE MODEL OF A MULTISIZE POLYPROPYLENE-FIBER-REINFORCED CONCRETE UNDER COMPRESSION

X. Yang,[1] N. Liang,[2,3*] X. R. Liu,[2,3] and Z. Zhong[2,3]

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To study the effect of multisize polypropylene fibers on the compression characteristics of concrete cubes, ten sets of polypropylene fiber-reinforced concrete test pieces were designed and fabricated to obtain their stress–strain curves and mechanical parameters at different ratios of coarse and fine fibers. Results for the cubes with multisize fibers were better than for those with to single-size ones. Based on test results, an improved statistical damage constitutive model for such a material is proposed.

1. Introduction

The addition of polypropylene fibers to concrete can significantly improve its mechanical characteristics, as many investigations have shown. Yin et al. [1] studied the postcracking properties of a polypropylene-fiber-reinforced concrete. Sukontasukkul et al. [2] examined the bending properties of a hybrid steel- and polypropylene-fiber geopolymer. Daneti et al. [3] studied the effect of polypropylene fibers on the shrinkage cracking properties of a light-weight-aggregate concrete. Li et al. [4] investigated the compression and shearing properties of a polypropylene-fiber-reinforced concrete and found that the addition of polypropylene fibers could considerably increase the toughness index and fracture energy of concrete. Koniki et al. [5] studied the effect of polypropylene-terylene hybrid fibers on the stress–strain curve of a high-strength concrete. Zhao et al. [6] examined the effect of polypropylene fibers on the durability of concrete. Results

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[1] School of Civil Engineering, Fujian university of technology, Fuzhou 350108, China
[2] School of Civil Engineering, Chongqing University, Chongqing 400045, China
[3] Key Laboratory of New Technology for Construction of Cities in Mountain Area (Chongqing University), Ministry of Education, Chongqing 400030, China
[*] Corresponding author; e-mail: cqulnh@163.com

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showed that the resistance of concrete to cyclic freezing and thawing increased with amount of the polypropylene fibers added. Aly et al. [7] investigated the effects of polypropylene fibers on the shrinkage cracking of concrete. Bagherzadeh et al. [8] studied the role of polypropylene fibers in improving the mechanical properties of concrete. Mastali et al. [9] examined the hardening properties of a polypropylene-fiber-reinforced concrete. Caggiano et al. [10] investigated the evolution trend of postpeak cracking of a steel- and polypropylene-fiber-reinforced concrete experimentally.

In our work, two or more different types of fibers were added to concrete to achieve performance advantages that cannot be obtained in the case of single-size fibers. Four types of fine and two types of coarse polypropylene fibers were used. Cube compression test were conducted on concretes with different-size fibers, and a one-parameter constitutive statistical damage model was used to analyze the effect of polypropylene fibers on the compression characteristics of concrete.

2. Constitutive Statistical Damage Model of Multisize Polypropylene-Fiber-Reinforced Concrete under Compression

2.1. Theoretical relations

In [11-12], it is shown that the Weibull distribution can be used to describe the internal damage of materials. Thus, the microunit strength of multisize polypropylene-fiber-reinforced concrete is assumed to obey the Weibull distribution. The distribution function of probability density is

$$P(\varepsilon_{\mathrm{p}}) = \frac{m}{\varepsilon_0}\left(\frac{\varepsilon_p}{\varepsilon_0}\right)^{m-1} \exp\left[-\left(\frac{\varepsilon_p}{\varepsilon_0}\right)^m\right], \tag{1}$$

where ε_p is the random distribution variable of microunit strength, and m and ε_0 are Weibull distribution parameters.

The damage degree D can be expressed as:

$$D = \int_0^F P(y)dy = 1 - \exp\left[-\left(\frac{F}{F_0}\right)^m\right]. \tag{2}$$

The microunit strength based on the Drucker–Prage principle is

$$F = \partial_0 I_1 + \sqrt{J_2}\ , \tag{3}$$

$$\alpha_0 = \frac{\sin\varphi}{\sqrt{9 + 3\sin^2\varphi}}\ , \tag{4}$$

where, φ is the internal friction angle of material; I_1 and J_2 are the first and second invariants of stress tensor.

In [13], it is shown that, in the one-dimensional stress state,

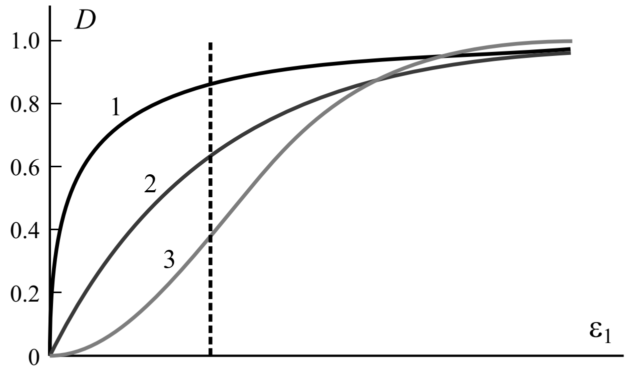
$$F = \left(\alpha_0 + \frac{1}{\sqrt{3}}\right)E\varepsilon_1\ , \tag{5}$$

where ε_1 is the axial strain.

Conjecturing that concrete obeys Hooke's law, we obtain that, according to the equivalent strain assumption,

$$\sigma_1 = E\varepsilon_1(1 - D)\ , \tag{6}$$

where σ_1 is the axial stress and E is the elastic modulus.



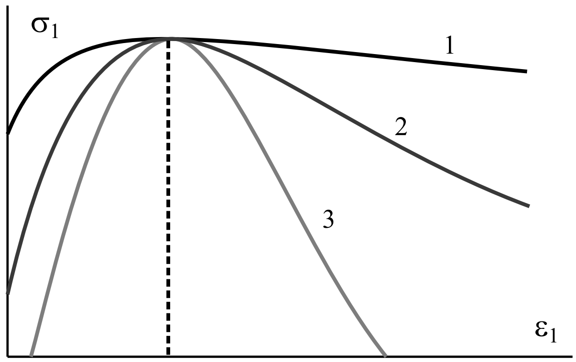


Fig. 1. Damage degree–strain curves D–ε_1 at m = 0.5 (1), 1.0 (2), and 2.0 (3).

Fig. 2. Stress–strain curves σ_1–ε_1 at m = 0.5 (1), 1.0 (2), and 2.0 (3).

Equation (2) is inserted into Eq. (6) to obtain that

$$\sigma_1 = E\varepsilon_1 \exp\left[-\left(\frac{F}{F_0}\right)^m\right], \tag{7}$$

$$\frac{d\sigma_1}{d\varepsilon_1} = 0 .$$

From Eq. (7), it follows that

$$F_0 = \left(\alpha_0 + \frac{1}{\sqrt{3}}\right) E\varepsilon_\mathrm{m} m^{1/m} . \tag{8}$$

Inserting Eqs. (5) and (8) into Eq. (2) gives that

$$D = 1 - \exp\left[-\frac{1}{m}\left(\frac{\varepsilon_1}{\varepsilon_m}\right)^m\right]. \tag{9}$$

Inserting Eq. (9) into Eq. (6) results in the constitutive statistical damage relationship

$$\sigma_1 = E\varepsilon_1 \exp\left[-\frac{1}{m}\left(\frac{\varepsilon_1}{\varepsilon_m}\right)^m\right]. \tag{10}$$

2.2. Physical significance and limitations on parameters of the one-parameter constitutive statistical damage model

Equation (10) has only one parameter, m . Figure 1 shows damage degree–strain curves D–ε_1 at different values of m . As can be seen, a smaller value of m leads to a sharper variation in the curve and to a greater value of D , and the critical damage degree corresponding to the peak strain point. The parameter m reflects the brittleness degree of material. The greater the of value m , the more brittle is the material, and the microunit distribution is more concentrated. Figure 2 shows stress–strain curves σ_1–ε_1 at different values of the parameter m . As can be seen, a smaller value of m leads to a higher postpeak strength and a more gentle slope of the postpeak stress–strain curve. A smaller value of the parameter m also led to a higher critical damage degree D in the prepeak stage and to a higher post-peak strength, which did not agree with some experimental results. The test pieces with a higher critical damage degree D in the prepeak stage

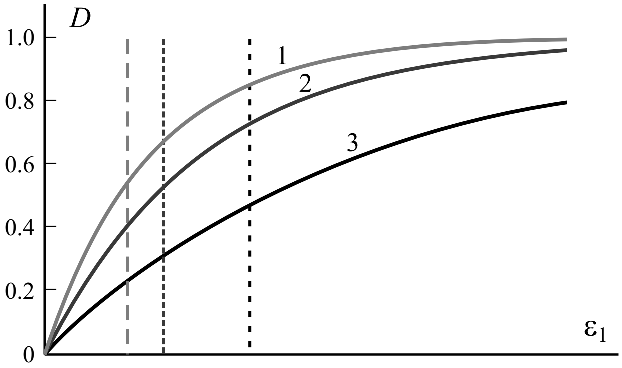


Fig. 3. Damage degree–strain curves D–ε_1 given by the two-parameter model at a = 0.5 (1), 1.0 (2), and 1.5 (3).

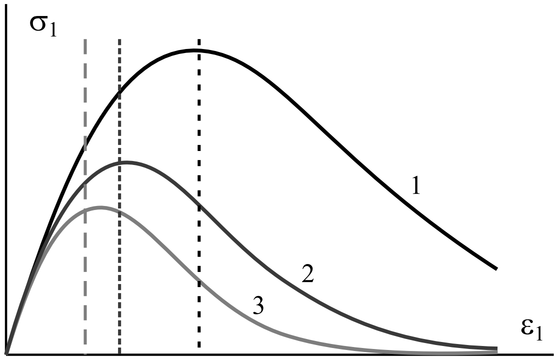


Fig. 4. Stress–strain curves σ_1–ε_1 given by the two-parameter model at a = 0.5 (1), 1.0 (2), and 1.5 (3).

not necessarily had a higher postpeak strength. This means that the one-parameter constitutive statistical damage model is limited and has to be improved.

2.3. Improved constitutive statistical damage model

Equation (10) was improved assuming that

$$\sigma_1 = E\varepsilon_1 \exp\left[-a\left(\frac{\varepsilon_1}{\varepsilon_m}\right)^m\right], \tag{11}$$

where a is an additional damage parameter, and the damage degree D is expressed as

$$D = 1 - \exp\left[-a\left(\frac{\varepsilon_1}{\varepsilon_m}\right)^m\right]. \tag{12}$$

Figure 3 presents damage degree–strain curves D–ε_1 in the case of different values of the parameter a. As can be seen, a larger value of parameter leads to a sharper variation in the curve, with the critical damage degree D corresponding to the peak strain point. The parameter a reflects the average macroscopic strength of the material. A smaller value of the parameter a leads to a higher average macroscopic strength. Figure 4 presents stress–strain curves σ_1–ε_1 for different values of the parameter a. As can be seen, a smaller value of a leads to a higher peak strength and peak strain of the material.

2.4. Fitting result of the improved constitutive statistical damage model

Table 1 lists the geometrical and mechanical characteristics of the olypropylene fiber used in tests. The cement was the common 42.5R cement produced by Chongqing Fuhuang Cement Co., Ltd. The coarse aggregate was gravel with a size of 5-10 mm.

The strength grade of the concrete used in the test was C30. According to the Standard for Test Methods for Fiber-reinforced Concrete [14], the internal dimension of each test mold was 100 × 100 × 100 mm. Ten experimental groups were prepared. No fiber was added to group A0, only plain concrete was used. A1-A4 were test pieces single-doped with

TABLE 1. Characteristics of Polypropylene Fibers

Fiber number	Diameter, mm	Length, mm	Tensile strength, MPa	Elastic modulus, GPa	Elongation at break, %	Density, g/cm^3	Recommended doping amount, kg/m^3
FF1	0.026	12	641	4.5	40	0.91	0.9
FF2	0.026	19	641	4.5	40	0.91	0.9
FF3	0.046	19	500	4.2	30	0.91	0.9
FF4	0.1	19	322	4.9	15	0.91	0.9
CF1	0.5	28	713	7.5	10	0.95	6.0
CF2	0.8	50	706	7.4	10	0.95	6.0

TABLE 2. Constituents of Polypropylene-Fiber-Reinforced Concretes (kg/m^3)

Test piece number	Fiber type	Cement	Sand	Gravel	Water	Doping amount of fiber	Sand ratio, %
A0	–	406	548	1221	207	0	23
A1	FF1	406	548	1221	207	0.9	23
A2	FF2	406	548	1221	207	0.9	23
A3	FF3	406	548	1221	207	0.9	23
A4	FF4	406	548	1221	207	0.9	23
A5	CF1	406	548	1221	207	6.0	23
A6	CF2	406	548	1221	207	6.0	23
A7	CF2+FF1	406	548	1221	207	5.1+0.9	23
A8	CF2+FF4	406	548	1221	207	5.1+0.9	23
A9	CF2+FF1+FF4	406	548	1221	207	5.1+0.45+0.45	23

TABLE 3. Fitting Result of the Improved Constitutive Statistical Damage Model

Test piece number	Elastic modulus, GPa	Parameter *a*	Parameter *m*	Fitting correlation coefficient
A0	26.088	0.7091	1.9972	0.9842
A1	34.111	0.9307	1.3891	0.9851
A2	32.878	0.8793	1.2521	0.9904
A3	32.308	1.0559	1.3795	0.9896
A4	30.993	0.7247	1.7261	0.9843
A5	29.175	0.6602	0.8627	0.9749
A6	28.503	0.8878	0.9945	0.9934
A7	30.657	0.5956	0.9355	0.9722
A8	34.307	1.1730	1.1493	0.9926
A9	32.729	0.6439	1.4579	0.9962

fine fibers of different sizes. According to the instruction manual provided by the polypropylene fiber manufacturer, the doping amount of fibers was 0.9 kg/m^3. A5-A6 were test pieces single-doped with coarse fibers of different sizes in amount of 6.0 kg/m^3. A7-A9 were hybrid fine and coarse fiber-reinforced test pieces. To guarantee the comparability of experiments, the total doping amount of coarse and fine fibers was 6.0 kg/m^3. Only the mixing ratio of the coarse and fine fiber was changed. Table 2 shows the mix proportion of concrete [15].

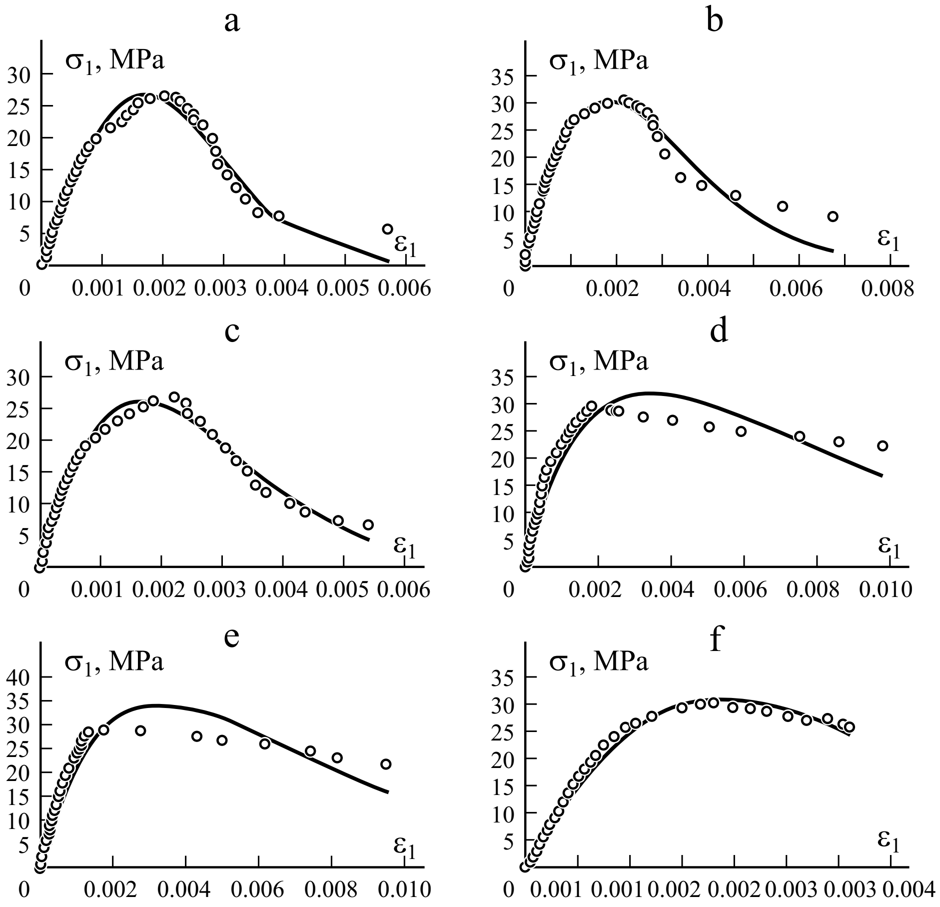


Fig. 5. Comparison of test data (○) and fitting results of the improved constitutive statistical damage model (——) for test pieces A0 (a), A1 (b), A3 (c), A5 (d), and A9 (f).

Figure 5 and Table 3 show fitting result of the improved constitutive statistical damage model. As can be seen, the A1-A9 concrete test pieces, with polypropylene fibers added, had a higher elastic modulus and lower value of m than the plain concrete test piece A0. This indicates that the addition of polypropylene fibers increases concrete rigidity and decreases its brittleness. In other words, its postpeak strength increases. The values of a of A1-A9 concrete test pieces exceeded that of A0 in the majority of cases. A smaller value of a indicates a higher average macroscopic strength of concrete test piece. Table 3 shows values of the elastic modulus and the parameters a and m for all test pieces. The fitting result of the improved constitutive statistical damage was good, and all correlation coefficients were exceeded 0.97 (see Table 4).

3. Correlation Coefficient Between Parameters of Multisize Polypropylene-Fiber-Reinforced Concrete

To determine the level of correlation between various parameters x and y of the reinforced concretes considered, the correlation coefficient

$$R(\mathrm{x}, y) = \frac{\operatorname{cov}(x, y)}{\sqrt{\operatorname{var}[x]\operatorname{var}[y]}} \tag{13}$$

was introduced, where cov(x, y) is the covariance of x and y. var[x] is the variance of x and var[y] is the variance of y. The results obtained are presented in Table 4.

TABLE 4. Correlation Coefficients

Correlation coefficient	Elastic modulus	Peak strength	Peak strain	*a*	*m*
Elastic modulus	–	0.5011	0.3944	0.5217	–0.2011
Peak strength	0.5011	–	0.1640	–0.1409	–0.017
Peak strain	0.3944	0.1640	–	0.8510	0.0069
a	0.5217	–0.1409	0.8510	–	–0.0701
m	–0.2011	–0.017	0.0069	-0.0701	–

A strong positive correlation, with $R = 0.851$, is seen to exist between the parameter a and the peak strain. But correlation coefficients between the parameter m and the peak strength, peak strain, and parameter a are small, with $R = –0.017$, 0.0069, and –0.0701, respectively. Changes in the parameter m also affect the prepeak strength, prepeak strain, and the critical damage degree D, but only slightly.

Conclusions

An improved two-parameter (m and n) constitutive statistical damage model of multisize polypropylene-fiber-reinforced concrete in compression has been introduced and investigated. The parameter m reflects the brittleness and a characterizes the macroscopic strength of the composite material. They were found by fitting calculation results to experimental data. From our investigation, the following conclusions were drawn.

1. The elastic modulus and peak strength of concrete reinforced with coarse and fine fibers were higher than those of concrete with only coarse fibers.

2. The parameter a and the peak strain had a positive correlation with a correlation coefficient of up to 0.851. Correlations between the parameter m and the peak strength, peak strain, and parameter were weak, with correlation coefficient smaller than 0.1.

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